HOLOGRAPHIC BOUND FROM SECOND LAW

JACOB D. BEKENSTEIN
The Racah Institute of Physics, Hebrew University of Jerusalem, Givat Ram, Jerusalem 91904, Israel
E-mail: bekenste@vms.huji.ac.il

The holographic bound that the entropy (log of number of quantum states) of a system is bounded from above by a quarter of the area of a circumscribing surface measured in Planck areas is widely regarded a desideratum of any fundamental theory, but some exceptions occur. By suitable black hole gedanken experiments I show that the bound follows from the generalized second law for two broad classes of isolated systems: generic weakly gravitating systems composed of many elementary particles, and quiescent, non-rotating strongly gravitating configurations well above Planck mass. These justify an early claim by Susskind.

1 What is the holographic bound ?

The holographic principle, first enunciated by 't Hooft,[1] claims the physical equivalence between pairs of physical theories. One member of a pair, T1, describes a bulk system $\mathcal{U}$ in a spacetime; the allied theory, T2, describes a boundary of that spacetime. For instance, string theory in the $D = 10$ spacetime $AdS_5 \times S^5$ is known to be equivalent to a supersymmetric gauge theory on the boundary. Some more examples are known, mostly in $D > 4$. Many regard the holographic principle as a criterion for good physics.

An obvious consistency requirement on the holographic principle is that the boundary of any system should be able to encode as much information as required to catalogue the quantum states of the bulk system: T2 should allow at least as many quantum states to reside on the boundary as T1 allows for the bulk (otherwise, so to speak, T2 does not know enough to be equivalent to T1). Suppose we go to $D = 4$ and take the logarithm of this inequality. On the one hand we get the entropy $\mathcal{S}$ of $\mathcal{U}$ and its gravitational field; on the other we get the entropy of the boundary, which analogy with black hole entropy suggests to quantify by one quarter of its 2-D area $\mathcal{A}$ in Planck units. Thus we have the guess (the holographic bound[1,2] here denoted HB; Planck units used throughout !)

$$\mathcal{S} \leq \mathcal{A}/4 \tag{1}$$

How do we know that this bound is true ?

As support for the HB, Susskind[2] described a *gedanken* experiment in which a system violating the HB is forced to collapse to a black hole by adding to it extra entropy–free matter. Susskind interprets the ensuing apparent violation of the generalized second law (GSL) as evidence that the envisaged system cannot really exist. In the clearer reformulation of Wald,[3] one imagines the system as a spherically symmetric one of radius R, energy E (with $R > 2E$, of course) and entropy $\mathcal{S}$ which violates the HB: $\mathcal{S} > \pi R^2$. A spherically symmetric and concentric shell of mass $R/2 - E$ is dropped on the system; by Birkhoff's theorem the total mass is now $R/2$. *If* the outermost surface of the shell reaches Schwarzschild radial coordinate

$r = R$, the system becomes a black hole of radius R and entropy $S_{BH} = \pi R^2$, which is lower than the original entropy $\mathcal{S}$! Susskind would conclude from this that, contrary to the assumption, the HB cannot be violated for $\mathcal{U}$. However, Wald points out an alternative conclusion: if $\mathcal{S} > \pi R^2$ the outcome of the process is not a black hole, e.g. the shell could bounce, or a naked singularity appear. Thus the Susskind argument is inconclusive.

Below I review two variations [4] on Susskind's argument which provide proof of the HB for a broad class of weakly gravitating systems and a broad class of strongly gravitating systems. I should mention that there do exist systems which violate the HB, e.g. a large enough spherical section of a flat Friedmann universe, or a collapsing sphere inside its gravitational radius. [5]

2 Systems with weak self–gravity

Consider first a weakly self–gravitating system $\mathcal{U}$ which may have arbitrary structure and constitution, but require it to be isolated to avoid the problem of the large section of the universe. Let $\mathcal{U}$'s proper mass–energy be E and its entropy $\mathcal{S}$. Further, imagine $\mathcal{U}$ is enclosed in a snug spherical box of radius R concentric with $\mathcal{U}$'s center of mass (c.m.) and let the box be made of entropy–less material. Any essential mass of the box is to be included in E. Weakly self–gravitating means $R \gg E$. Assume in addition that $R \gg 1/E$: the system is far from being an elementary particle, and thus large compared to its own Compton length. These assumptions together imply that $R \gg 1$ (we always deal with systems large on Planck length scale). Now imagine dropping the sphere freely into a Schwarzschild black hole of mass $M = R^2(8E)^{-1}$ (assuming, of course that it is possible to find black holes of any mass). Now because $R \gg E$, $M \gg E$ and $2M \gg R$. Thus $\mathcal{U}$ constitutes a small perturbation on the black hole and is very small compared to it; there is thus no reason why the black hole should be destroyed by its infall or why $\mathcal{U}$ should be torn up by tidal forces.

Now if $\mathcal{U}$ falls from rest at some point far from the hole, its energy as measured at infinity is $< E$; writing $\mathcal{E}$ for the energy lost by $\mathcal{U}$ and black hole to radiation (see below) during the infall, we see that the hole increases its mass by $< E - \mathcal{E}$. The black hole's initial entropy, $S_{BH} = 4\pi M^2$, thus increases by $\Delta S_{BH} < 8\pi M(E - \mathcal{E})[1 + \frac{1}{2}(E - \mathcal{E})/M]$. By the GSL, ΔS_{BH} plus the radiations entropy, $S_{\rm rad}$, must be at least as large as $\mathcal{S}$, the entropy which disappears from sight. We thus have

$$\mathcal{S} < 8\pi M(E - \mathcal{E})[1 + \frac{1}{2}(E - \mathcal{E})/M] + S_{\rm rad}. \tag{2}$$

Now the energy lost by the infalling $\mathcal{U}$ to gravitational radiation is known [6] to be of $O(E^2/M)$, here reducing to $O\big(E(E/R)^2\big)$. Because $R \gg E$ we may neglect this contribution to $\mathcal{E}$ in the last inequality in comparison with the much bigger E. The other contribution to $\mathcal{E}$ is by Hawking radiance of the hole. One can approximate the power so emitted by that of a sphere with radius $2M$ radiating according to the Stefan–Boltzmann law at the Hawking temperature $T_H = (8\pi M)^{-1}$. Thus the rate of black hole mass change is

$$\dot{M} \approx -\frac{N}{15360\pi M^2}, \tag{3}$$

where N denotes the effective number of radiated species. If $\mathcal{U}$ was dropped at Schwarzschild time $t = 0$ when its center of mass was at $r = \alpha 2M$ ($\alpha \gg 1$), then integration of the geodesic equation describing $\mathcal{U}$'s c.m. motion in the Schwarzschild metric shows that the *bottom* of the sphere reaches the horizon $r = 2M$ (and the sphere's center reaches *proper* height R above it), and the infall is finished for all practical purposes, at time

$$t \approx 4M \left[\frac{1}{3}\alpha^{3/2} - \frac{1}{4}\ln(R/4M) + O(\alpha^{1/2})\right]. \tag{4}$$

The textbook statement that a particle takes an infinite t time to reach the horizon is recovered from the $R \to 0$ limit here; but obviously that eternity is only logarithmically big !

The Hawking energy emitted during the infall to the black hole of mass $M = R^2(8E)^{-1}$ is thus $\mathcal{E} = |\dot{M}| \times t \approx N(480\pi R^2)^{-1}\left[\frac{1}{3}\alpha^{3/2} - \frac{1}{4}\ln(2E/R)\right] E$. Because $R \gg 1$, the factor $N(480\pi R^2)^{-1}$ is very small compared to unity given that in nature $N = O(10)$. In fact this small factor easily outbalances any moderately large $\alpha^{3/2}$ and the logarithmic factor which is never big. Thus $\mathcal{E} \ll E$, and hence it is justified to drop $\mathcal{E}$ everywhere in (2). Also, because $E \ll R \ll 2M$ we can drop the E/M correction in Eq. (2). The last two inequalities also justify *a posteriori* our use of a Schwarzschild metric of mass M for calculating the time interval.

What about $S_{\rm rad}$ in Eq. (2) ? The infall gravitational radiation is coherent and so should carry negligible entropy, particularly since its energy is small. The Hawking radiance entropy is $\sim \mathcal{E}/T_H = 8\pi M\mathcal{E}$, obviously negligible compared to the dominant term, $8\pi ME$, in Eq. (2). Putting $M = R^2(8\mathcal{E})^{-1}$ in that formula we get $\mathcal{S} < \pi R^2$, which is precisely the HB, Eq. (1).

The above treatment presumes that radiation pressure from the Hawking's radiance does not prevent $\mathcal{U}$ from reaching the horizon and does not drastically prolong the infall time estimated in Eq. (4). Are these suppositions true ? The momentum *flux* from Hawking radiance at radial coordinate r is obviously $|\dot{M}|(4\pi r^2)^{-1}$. Assuming that all the radiation hitting the sphere is absorbed, the rate at which it picks up momentum from the radiation, in its own rest frame (four velocity u^β), is at least this flux times the sphere's crossection [which is $O(\pi R^2)$], times $(dt/d\tau)^2$, where τ is the sphere's proper time (if the sphere reflects radiation, one must multiply this result by a factor between 1 and 2 to account for backscattering). One factor $dt/d\tau$ accounts for the blueshift of the momenta of the Hawking quanta perceived in the falling sphere's frame; the second corrects for the faster arrival of quanta due to the time dilation and gravitational redshift. Since the sphere falls from $r \gg 2M$, we have $dt/d\tau \approx (1 - 2M/r)^{-1}$. Putting all the factors together with Eq. (3) and dividing by E, we find the acceleration of the falling sphere measured in its own frame (its acceleration scalar):

$$a \approx \frac{NR^2}{61440M^2Er^2}\frac{1}{(1-2M/r)^2} \tag{5}$$

Is this big or small ? The quantity to compare it with is the acceleration scalar

of a stationary point at radial coordinate r, $g = Mr^{-2}(1-2M/r)^{-1/2}$:

$$\frac{a}{g} = \frac{N/E}{7680R}\left[\frac{R}{2M(1-2M/r)^{1/2}}\right]^3. \tag{6}$$

Since we assumed $R \gg 1/E$ and have $R \ll 2M$ in our *gedanken* experiment, it is apparent that if the sphere is not very close to the black hole's horizon, the radiation pressure acceleration is negligible negligible on the natural scale g. And because $R \ll 2M$, the factor in square brackets in Eq. (6) only grows to 2 when the bottom of the sphere touches the horizon. Hence the radiation pressure deceleration is totally negligible throughout. We conclude that $\mathcal{U}$'s c.m. does move accurately on a timelike geodesic all the way down to the horizon, as has been assumed all along. Smallness of a/g also means that it is unnecessary to correct for quantum buoyancy effects, as is the case when the system is suspended.[7]

3 Systems with strong self–gravity

The argument in Sec. 2 fails when the system's self–gravity becomes strong, say, $R < 20E$ because in this case, $M = R^2(8E)^{-1}$ implies that $2M < 5R$ so we cannot be sure that $\mathcal{U}$ is not tidally torn up by the black hole. In addition we see that $M < 50E$ so that $\mathcal{U}$ is a significant perturbation on the hole. This means we cannot assume it will fall on a geodesic of the background metric, or even that it will not cause a singularity on the horizon. We thus change our strategy; we now employ a small black hole, denoted $\mathcal{H}$ below, whose task is to catalyze the conversion of the strongly gravitating system, now denoted $\mathcal{V}$, into a single large black hole.

As before we assume $\mathcal{V}$ is isolated, which means its exterior geometry is asymptotically flat; this makes it straightforward to define its total energy E. To simplify the deductions we assume that $\mathcal{V}$ is quiescent and nonrotating, i.e. that its geometry is nearly static. And finally we shall assume that $E \gg \mathrm{Max}(\mu^{-1}, 10^3\sqrt{N})$ where μ is the mass of the lightest massive elementary constituent of $\mathcal{V}$ and N, again, is the number of species in radiation. These restrictions will guarantee compatibility of the bounds that we shall assume presently on $\mathcal{H}$'s mass. Note that $\mathcal{V}$ is required to be very massive on Planck scale.

Now, it is pretty clear that the area $\mathcal{A}$ of a 2-D surface enclosing $\mathcal{V}$ must exceed $4\pi(2E)^2$ for otherwise $\mathcal{V}$ would already be a black hole (there are pathological surfaces which can be smaller; see Ref. 4 for a cleaner definition). And it must exceed it substantially; otherwise $\mathcal{V}$ would be unstable against black hole formation, i.e., not quiescent. We thus take it that $\mathcal{A}$ is substantially greater than $16\pi E^2$.

Let us enclose $\mathcal{V}$ in a roomy quasispherical box concentric with its c.m. whose walls are robust enough to trap all radiation that will be produced, save for gravitational waves which penetrate almost every barrier. We take the inner radius of the box to be $r \approx 2\cdot 10^2 E$ (in the sense of approximate Schwarzschild coordinates). $\mathcal{V}$ being strongly gravitating, it evidently occupies only a small central region of the box; therefore, most of the box volume is close to flat spacetime. The box is likely to be massive, and so to cause a shift in the standards of time and energy within it as compared with the outside world. All statements about these quantities refer to the interior.

Let $\mathcal{H}$'s mass m be restricted by $\mathrm{Max}(\mu^{-1}, 10N^{1/5}E^{3/5}) \ll m \ll E$. The upper and lower bound here are consistent by virtue of the constraints on E we adopted. We drop the small Schwarzschild black hole $\mathcal{H}$ from rest from within the box and near its wall and with no angular momentum. Because $\mathcal{V}$ is strongly gravitating but non–rotating, it will be almost spherical and so we may roughly approximate its exterior metric by Schwarzschild's. Then by Eq. (4), $\mathcal{H}$ takes an (infall) time $t_f \sim 10^3 E$ to reach $\mathcal{V}$. This does not mean $\mathcal{H}$ stops within $\mathcal{V}$; the latter may be tenuous enough to allow $\mathcal{H}$ to cross it, rise within the box to near the wall, and turn around for another such cycle. In one extreme case the black hole is trapped in $\mathcal{V}$ after one or two such cycles; in the other the trapping time is maximal, t_t, and corresponds to the number of passes through $\mathcal{V}$ that $\mathcal{H}$ must make in order for its crossection $16\pi m^2$ to sweep through the whole volume of $\mathcal{V}$, thus insuring a hard collision with the more massive system. Because the linear extent of $\mathcal{V}$ is of order $(\mathcal{A}/4\pi)^{1/2}$, it takes some $(E/m)^2$ passes to do this, assuming, in harmony with the fact we did not take $\mathcal{V}$ to be spherically symmetric, that each pass through it is in a different direction. Thus $t_t \sim 10^3(E/m)^2 E$.

Once trapped in $\mathcal{V}$, the little hole begins to digest its host. Since $\mathcal{V}$ is strongly self–gravitating, $\mathcal{H}$ will at first move at relativistic speed so that accretion may be inefficient. But collisions with $\mathcal{V}$'s components will slow it down (which is entirely possible if it settles down deep in the gravitational well of $\mathcal{V}$), and allow the accretion to speed up. Aided by its tidal field, $\mathcal{H}$ can tear up parts of $\mathcal{V}$ to smaller pieces, down to the scale of elementary constituents. Since the largest Compton length of these is smaller than $\mathcal{H}$'s size ($m \gg \mu^{-1}$), the hole can swallow anything that comes within its reach. The duration of this digestion stage obviously depends on details about $\mathcal{V}$ but there should be an upper bound to it, t_d, which should be a function of only the important scales of the problem E and m. Were m comparable with E, the digestion stage would obviously be over in the crossing time $\sim E$. For small m/E the accretion effectiveness should scale as $\mathcal{H}$'s crossection m^2. Thus on dimensional grounds we guess $t_d \sim (E/m)^2 E$. Towards the end of the digestion the joint system $\mathcal{V} + \mathcal{H}$ is likely to evolve rapidly. Fragments that get ejected from $\mathcal{V}$ as it is being swallowed remain trapped in the box, and will eventually fall back onto $\mathcal{H}$, be broken up and swallowed. This last stage should last a time $\sim E$ because it obviously involves an instability, instabilities grow exponentially, and our two scales, E and m, are merging as the black hole ingests all of $\mathcal{V}$. We thus see that the digestion is likely briefer than the trapping, no matter how big N.

But will not $\mathcal{H}$ Hawking evaporate before it consumes $\mathcal{V}$? The Hawking evaporation timescale of the initial black hole is, according to Eq. (3), $t_H \approx 5120\pi m^3/N$. But since we assume $m \gg 10N^{1/5}E^{3/5}$, it follows that $t_H \gg 10^9(E/m)^2 E$, so over the course of the trapping and digestion times, $\mathcal{H}$ hardly loses mass by Hawking emission. Of course as $\mathcal{H}$ gets more massive by accretion, the Hawking emission weakens even further: Hawking radiance is negligible in our *gedanken* experiment.

Obviously the accretion will generate heat, and radiation (e.g. electromagnetic but little gravitational because of its poor coupling to matter) will thus leak out of $\mathcal{V}$ into the box, which we have assumed can trap the nongravitational quanta. As it nears its end, $\mathcal{V}$ should approach spherical symmetry because it is nonrotating, and the elastic forces that could keep it aspherical will succumb to gravitation as the

black hole gnaws its way through it. Finally, as the black hole finishes its meal, it recovers its original Schwarzschild form, and establishes thermodynamic equilibrium with the radiation filling the box. In the presence of a spherically symmetric system at its center, the box can be regarded as perfectly spherical and thus gravitationally irrelevant (apart from the gravitational redshift it induces inside it).

How long does it take for the cavity to reach equilibrium with $\mathcal{H}$? An upper bound on this equilibration time t_{eq} is set by the time the final hole would take to fill the cavity just with Hawking radiation (we saw that the evaporation time of the faster radiating initial hole is long compared to the digestion time which is also the heating time). It shall transpire that the final hole has a mass very near E. Thus its Hawking temperature (and the temperature of the equilibrated radiation) is $T = (8\pi E)^{-1}$. Using the volume $(4\pi/3)(2 \cdot 10^2 E)^3$ for the box's interior and Boltzmann's energy density for black body radiation, we compute the energy of radiation trapped in the box as $\mathcal{E} \approx 55NE^{-1}$. Dividing this by the Hawking power (3) with $M \to E$ gives $t_{eq} < 3 \cdot 10^6 E$. Since we assume $E \gg m$, t_{eq} may even be shorter than t_d or t_t.

It might be claimed that not t_{eq}, but the relaxation time t_r for the radiation in the box is the informative timescale: if the radiation and hole are slightly out of equilibrium, how long do they take to reach it or return to it ? Intuitively this time should be of order of the times for absorption and reemission of a typical quantum of radiation by $\mathcal{H}$; in or near equilibrium these two equivalent. Now a typical quantum not very close to $\mathcal{H}$ will be absorbed in its next crossing of the box with probability roughly equal to the solid angle subtended by the hole at the quantum's starting point divided by 4π. This is about $(10^{-2})^2$ for our cavity. Dividing the light crossing time $2 \cdot 10^2 E$ by this probability gives the estimate $t_r \sim 2 \cdot 10^6 E$ which is very like the upper bound on t_{eq}. Therefore, $\mathcal{H}$ comes into precise thermodynamic equilibrium with the cavity's radiation in a time no longer than it took to digest $\mathcal{V}$.

Making use twice of our assumption $E \gg 10^3\sqrt{N}$ in our previous expression for $\mathcal{E}$, we get that the equilibrium radiation energy in the box is $\mathcal{E} \approx 55NE^{-1} \ll 10^{-4}E$. And since $m \ll E$ the energy of the final black hole is very near E. But what about the energy $\mathcal{E}_g$ carried away by Hawking gravitons ? In its initial stages the black hole's power in gravitons is given by Eq. (3) with $N = 1$ and $M \to m$. Multiplying this by, say, the trapping time t_t gives $\mathcal{E}_g \sim 0.02E^3m^{-4} \ll 2 \cdot 10^{-7}EN^{-1}$ in view of our restriction $m \gg 10N^{1/5}E^{3/5}$ and $E \gg m$. In its latter stages the hole's graviton power is again (3) with $N = 1$ but this time with $M \to E$. Over the relaxation time t_r this gives energy $\mathcal{E}_g \sim 10^2E^{-1} \ll 10^{-4}EN^{-1}$, where we have twice used our assumption $E \gg 10^3\sqrt{N}$. In summary, the energy lost to gravitons from the dropping of $\mathcal{H}$ into $\mathcal{V}$ through the equilibration of the final hole with the cavity is $\mathcal{E}_g \ll 10^{-4}EN^{-1}$. The upshot of this paragraph is that it was self–consistent to assume that in its final state $\mathcal{H}$ has energy very nearly E and Hawking temperature very nearly $(8\pi E)^{-1}$.

We now draw up the balance of entropy. Originally we had entropy $\mathcal{S}$ in $\mathcal{V}$ and $S_{BH} = 4\pi m^2$ in the initial $\mathcal{H}$. Just after its equilibration and relaxation, the cavity radiation has entropy $\frac{4}{3}\mathcal{E}T^{-1} = \frac{32}{3}\pi E\mathcal{E}$ (the $\frac{4}{3}$ is well known from Boltzmann's formulae for black body radiation). Hawking gravitons have carried away entropy $S_g \sim \mathcal{E}_g T_H^{-1} \ll 2 \cdot 10^{-7} \cdot 8\pi mEN^{-1}$ in the digestion stage and $S_g \sim \mathcal{E}_g T^{-1} \ll$

$10^{-4} \cdot 8\pi E^2 N^{-1}$ in the equilibration stage. The final $\mathcal{H}$ has entropy $S'_{BH} = 4\pi(E + m - \mathcal{E} - \mathcal{E}_g)^2$. Applying the GSL and rearranging terms gives us

$$\mathcal{S} < 4\pi E^2 \left[1 + \frac{2m}{E} + \tfrac{2}{3}\frac{\mathcal{E}}{E} + \frac{2\mathcal{E}_g}{E} + \frac{S_g}{4\pi E^2} + \cdots \right] \tag{7}$$

where the ellipsis denotes second order terms like $m\mathcal{E}E^{-2}$ or ${\mathcal{E}_g}^2 E^{-2}$. Let us look at the corrections to "1" in the square brackets. We have just seen that in the early stages of $\mathcal{V}$'s digestion $\mathcal{E}_g E^{-1} \ll 2\cdot 10^{-7} N^{-1}$ and $S_g(4\pi E^2)^{-1} \ll 4\cdot 10^{-7}(m/E)N^{-1}$, while in the latter stages $\mathcal{E}_g E^{-1} \ll 10^{-4} N^{-1}$ and $S_g(4\pi E^2)^{-1} \ll 2 \cdot 10^{-4} N^{-1}$; in addition, $\mathcal{E}E^{-1} \ll 10^{-4}$. Hence the corrections to "1" are tiny (we assumed all along that $m \ll E$). On the other hand, we have pointed out that the area $\mathcal{A}$ of a 2-D surface surrounding the original system $\mathcal{V}$ must substantially exceed $16\pi E^2$. It follows from inequality (7) that $\mathcal{S}$ of the original system obeyed the holographic bound (1).

4 Summary and Caveats

We have thus vindicated Susskind's idea that the GSL can serve as the basis of a proof of the HB, albeit a proof for restricted classes of systems. The present account leaves out a number of details treated in the original paper.[4] Beyond that one can imagine situations which would sidestep the proof in Sec. 3. For example, were $\mathcal{V}$ spherically symmetric save for a straight "tunnel" though its center, then by dropping $\mathcal{H}$ down the tunnel one could arrange for $\mathcal{H}$ to go back and forth without ever colliding with anything in $\mathcal{V}$. This evidently can be fixed by dropping $\mathcal{H}$ in a generic direction. Or one could fear that as a result of random photon emission by the half eaten $\mathcal{V}$, the final $\mathcal{H}$ could be left with net momentum with respect to the box, so that it could collide with and destroy its wall. Again, a detailed calculation shows the timescale for the envisaged motion is much longer than t_d or t_r, so if the experiment is done as soon as possible, no problem occurs.

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